OUR HOMES ARE MUSEUMS
GEOMETRICAL PRINCIPLES IN THE DESIGN OF OBJECTS

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Our aim in this talk is to show how geometrical principles play a fundamental role in the design of objects made for our homes. Geometrical shapes and transformations, as well as ingenuity, give a methodological approach to the rational design process. As a consequence our homes are design museums and this has strong consequences for teaching and learning mathematics. We will see why and how.

Keywords: *Geometry, design, teaching geometry.*

1. INTRODUCTION

Our aim in this presentation is to face the problem of the use of geometry in the design of objects and to derive from our analysis some interesting didactical consequences for the teaching and learning of three dimensional geometry.

From the very beginning we assume that design is the result of a complex procedure because it needs to solve, at the same time, many requirements of different nature: economical restrictions, sociological behaviours, fashions, marketing promotions, durability, transportability, etc. Geometry contributes to design but it is just one component of it and it does not guarantee ideal solutions. Nevertheless, Geometry introduces interesting solutions.

If we define design as the creative process which make possible a posteriori the real production of an object then, clearly, geometry becomes a useful tool for thinking three-dimensional shapes and for representing the ideas by means of drawings or models. The design is a project and this distinguishes it from the traditional productions by artists, stonecutters, ceramicists, ironworkers, gardeners, etc. Making a bread is an important practical issue but it does not involve a previous design development.

From this point of view the design has an old tradition in Architecture and a short history in industry. During the XXth Century, names like Walter Gropius (Bauhaus), Lászlo Moholy-Nagy (New Bauhaus in Chicago), the people in the Hochschale für Gestaltung (Ulm), Alvar Aalto, Frank Lloyd Wright, Ludwing Mies van der Rohe, Raymond Loewy, Max Bill, Shiro Kuramata, Hans Gugelot, Otl Aicher, Michael Graves and many other (Ch. Fiell and Peter Field, 2001) have made possible powerful results either at the theoretical level or at the production process.

2. GEOMETRICAL PRINCIPLES IN DESIGN

We have identified six generic principles which clarify the possible contribution of Geometry to Design.

Principle 1. Geometry facilitates folding solutions.

Many objects, from fans to umbrellas, from seats to tables or beds need to have some kind of folding procedures (e.g., for transportability reasons or for storing purposes). Then the use of isometries and basic shapes give interesting solutions.

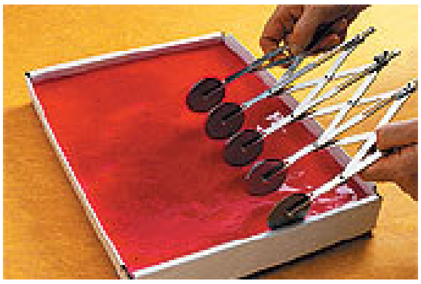


Figure 1. Cake divider

Principle 2. Geometry is a must in multi-functional designs

To achieve the good design of objects that may be useful for several purposes, e.g., a seat which becomes a table, one needs to use appropriate shapes and transformations (isometries mainly).

Principle 3. Geometrical packing orientates packaging problems

Packaging requires to provide many useful characteristics, e.g., minimal boxing measures, transportability, to store, to lay up, minimal cost, etc. Geometrical forms (cubes, boxes, tethahedra,…) give suitable solutions.

Principle 4. Geometrical measurements, proportions and shapes are basic in ergonomic design

Many designs are made at the service of people. Statistical characteristics of the population, measures that would satisfy to 95% of possible users, appropriate proportions of the object which reflect human's proportions (¡seats!) and shapes adequate to human bodies… all these issues are essential components of ergonomic design.

Figure 2. Hats for all (in Vietnam)

Principle 5. Geometrical transformations play a role in the design process as well as in the functional characteristics of many tools

As we have stated this principle it is clear that the classical transformations in the plane or in the space play this double role. On one hand designers may use a given transformation in the process of their projects (e.g. making a giant cup of tea by applying a similitude to a standard cup). But what really is important is that many actions to be performed by designed tools correspond to geometrical transformations (e.g. to open a wine bottle by means of a corkscrew is to perform two helicoidal movements). Next table sums up a list of basic transformations and objects related to them.

Transformations	**Examples of design of everyday life objectes**
Translations	Telescopic stairs; Repetition of seats; friezes, screens;…
Rotations	Can's opener; mut-cracker; clocks; folding table; lamps…
Symmetries	Left-hand designs; shoes; scissors; folding forms;
Helicoidal transformations	Bottle's opener; screws; folding forms; steps;…
Similitudes	Beach umbrellas; lamps, decorative fans; models;…
Perspectives	Photographic machines; space arrangements
Twinning, intersecting,…	Location of designs inside other designs
Dissections	Tangrams; tables (re-arrangements);…
Code's transformation	Hotel keys; code bars in packaging,…

Figure 3

Principle 6. Geometrical shapes give many times optimal solutions

By this principle we mean the use of well-known geometrical figures to give forms to objects, e.g., polyhedra, polygons, basic curves, quadric surfaces, etc. The following samples cover the most popular uses (Alsina and Nelsen, 2006):

Polyhedra	**Everyday life objects**
Cubes	Standard dice, bouillon cubes, Rubik's cube, boxes, etc.
Tetrahedra	3D puzzles, tripods, tetrahedral dice
Octahedra	Mineral crystals, octahedral dice
Dodecahedra	Paperweight, desk calendar, dodecahedral dice
Icosahedra	MAA logo, icosahedral dice, domes, sculptures
Prisms	Toblerone™ package, candy boxes, pencils
Pyramids	Egyptian pyramids, top of an obelisk, plumb bob
Bipyramids	Toy tops, jewels
Other polyhedra	Jewels and jewelry, soccer balls, puzzles

Polygons	**Everyday life objects**
Triangles	Traffic yield sign, danger sign, musical instrument
Quadrilaterals	Sheets of paper or cardboard, tiles
Pentagons	Chrysler logo, tables, Department of Defense building, paper strip tied into an overhand trot
Hexagons	Tiles, plates, cross-section of a pencil, hex bolts and nuts

Octagons	Stop sign, trays, tables
N-GONS	Some clock, faces, foreign coins
Star gons	Star fish, Star of David
Curve	**Everyday life objects**
Circle	Rimof plate or glass, coin, wheel, ring
Ellipse	Liquid in a inclined glass, circle viewed at an angle
Parabola	Cable on a suspension bridge
Hyperbola	Pofile of some bells, six arcs at the point of a sharpened pencil
Sine curve	Snake's path, sea waves
Cycloid	Trajectory of a point on a wheel
Catenary	Power lines, hanging chain
Spiral	Grooves in vinyl record or CD, tape in a cassette, coiled rope

Quadric surfaces	**Examples in design of everyday life objects**
Circular cylinders	Radio Ekco AD65 of Wells Coats (1934) for E.K. Cole Cylindra-Line Collection of Arne Jacobsen (1967) for Skelton. Pencis, water glasses, containers, bottles, paper rolls; tubes, hats,…
Circular cones	Il conico (tea pot) Aldo Rossi (1988) for Abssi Ice-cream cones; glasswares; road markers; hats; frackages,...
Ellipsoids and spheres	Il Ballitore of Richard Sapper (1983) for Alessi; Balls (rugby, soccer, ping-pong,…); ornamental balls; boxes; meat boilers; hats;…
Paraboloids of revolution	Collator of Max de Chinois (1990) for Alessi; TV antennas; headlamps for cars;…
Hyperboloids of 1 sheet	Bells; baskets; drums; metallic musical instruments,…
Hyperbolic paraboloids	Seats on horses; park structures; fried potatoes,…

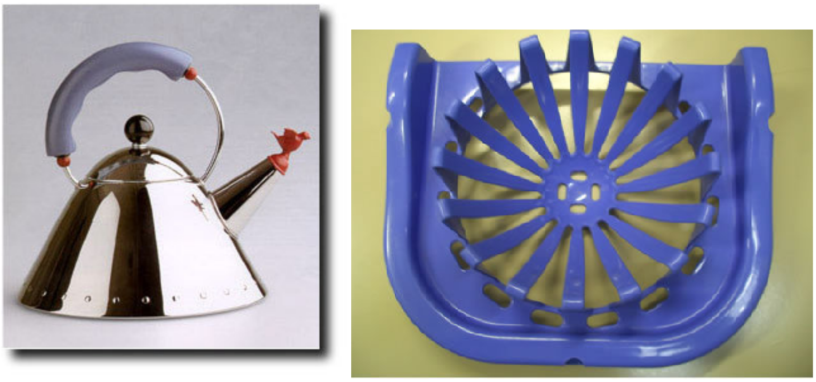
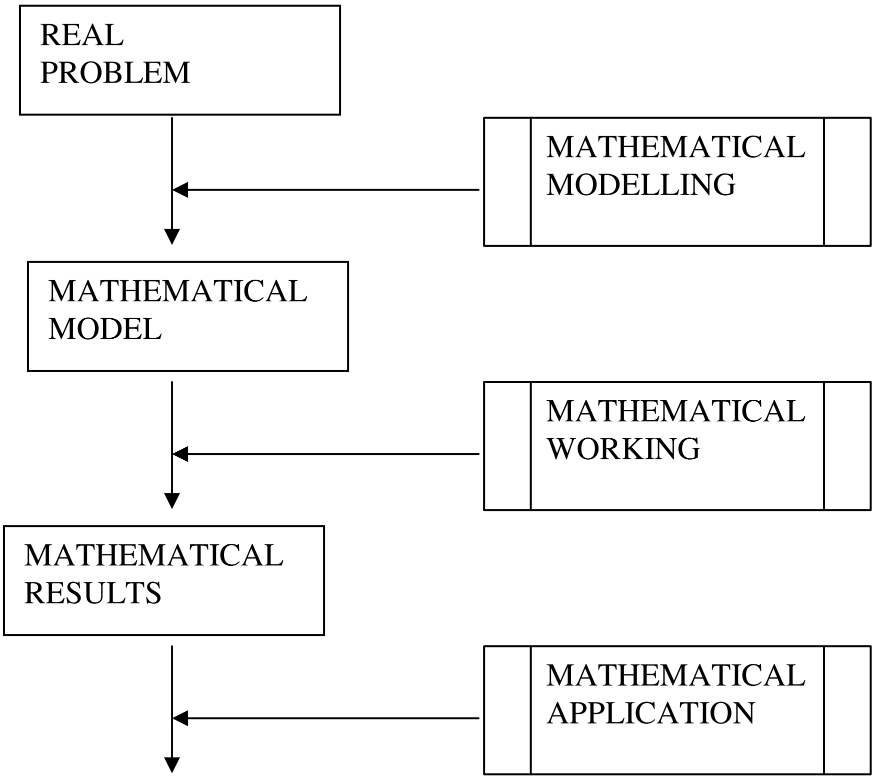


Figure 4. a, b

3. DESIGNS AND TEACHING GOALS

In the last years I have been proposing teachers to be aware of the designs we have at home as a way to show how important is mathematics in our everyday life (*motivation*) and how fruitful could be to use these home materials as hands-on objects in the classroom (*learning opportunities*).

As a mathematic educator I am convinced that teaching via *modelling and applications* is an interesting option. The classical diagram shows the steps for combining modelling-working and applying.



PRACTICAL CONSEQUENCES

One major challenge in mathematics education is to achieve the goal that realistic modelling and realistic application be indeed implemented in courses. This, however, is not so easy. We have clear evidence that there are many difficulties entailed in introducing this approach in our everyday teaching.

With all these designs that we have at home we can appreciate interesting applications of geometry but at the same time we can induce "problem posing" activities as well as "problem solving". The following are good examples of rich tasks:

(a) Take glasses of wine and cups. For any one conjecture the relation between the altitude and the perimeter of the superior cercle. Then make appropriate measures (John Mason).
(b) How can you distribute holes in a sphere in a uniform way?
(c) Close one eye and mark on a mirror the profile of your face. Which measures will have this representation?
(d) How to divide a rectangular cake (with chocolate icing on top and all the sides) between five people in a fair way?

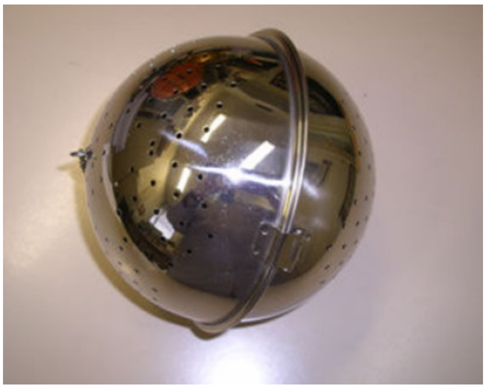


Figure 5

We would like to make some remarks (see (Alsina, 2007)) concerning the problems to face if we want to integrate designs in the mathematical classrooms:

We need teachers who are confident

The main objection is that pre-service education does not provide teachers enough knowledge and experiences to be confident in dealing with applications and modelling. Thus many improvements on pre-service and in-service training need to be made.

The level of learners must orientate the choice

Clearly, the level of learners will orientate us as to which choice to make concerning applications. Applications (and specially research activities on them) are ideal for inducing cooperative work or teamwork. Good assessment, e.g. in setting up projects, needs to take into account individual and cooperative aspects. Preparation for cooperative work is a crucial goal in today's circumstances.

The applied approach is time consuming

Many teachers do not organize active learning visits, experiments, etc. because there is a lack of time. It seems there is no problem of time for chalkboard expositions. What is true is that our proposals imply a careful preparation of agendas.

Technology is not the solution

The growing power of technologies may induce some people to believe that these new devices are the essential tools for providing support for well-structured experiences; that simulations and images may be enough to eliminate completely the need for "real experiments" and hands-on materials. This is not possible. New software gives new mathematical insights but can't replace "learning by making". In our discussion here, **technology serves to complement** what we are presenting.

Hands-on materials are always available

While textbooks and classroom materials are produced, with a wide range of alternative, very few kites can be purchased in the market. But many materials can be easily made and real objects are everywhere. Fortunately, many "free" opportunities exist around us. Reality is free.

Realistic activities need to be properly integrated

There is the risk that realistic experiences can become isolated, like islands in the sea of formal instruction. Such experiences are not useful if they are not combined with the usual teaching practice. These actions serve to assist students to learn important concepts, and of giving them new opportunities to develop new skills.

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